1

Cop = B
Motorist A

starting pt.
$\downarrow$

$B \rightarrow a$

$A \rightarrow V_0$

a) When the Cop catches the motorist, both their distance from starting point are same.

$B \Rightarrow \quad x = V_0 t$

$A \Rightarrow \quad x = \frac{1}{2} a t^2$

$\Rightarrow \quad V_0 t = \frac{1}{2} a t^2$

$= t = \cancel{\sqrt{\frac{2V_0}{a}}} \; \frac{2V_0}{a}.$

time when they catch up.

$\text{dist} = \frac{1}{2} a \times t^2 = \frac{1}{2} a \left(\frac{4V_0^2}{a^2} \right).$

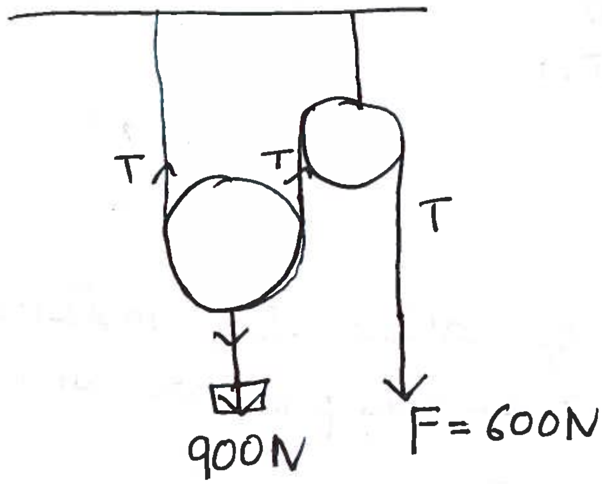
$= \boxed{2V_0^2 / a}$

b). Speed of officer the moment he catches.

$V = U + at.$

$= 0 + a \times \frac{2V_0}{a} = \boxed{2V_0}$

(2)



Weight = Mass x g

900 = M x g

90kg = M

T = F = 600

a)

$Ma = 2T - 900$

$$\boxed{a = \frac{300}{90} = 3.3}$$

b). If F moves by 'd' then mass goes up by d/2.

So if mass goes up by 1m, F would go by 2m.

Work = F.d = 600 · 2 = $\boxed{1200}$

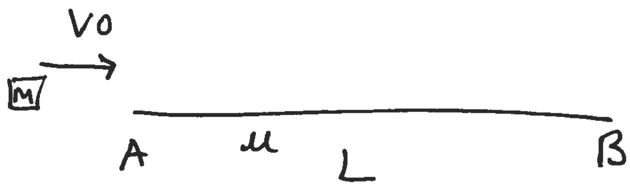
c) Force required for keeping the weight at rest

means a = 0.

$2T = 900$

$$\boxed{T = 450 = F}$$

3



M stops at B.

b). ~~Ae~~ ~~Bn~~ $L = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2L}{a}}$.

$v^2 = u^2 + 2as$.

$= 0 = V_0^2 + 2aL \Rightarrow \boxed{a = \frac{-V_0^2}{2L}}$

c) friction = μmg

$\mu mg = -ma$. $\quad a = -\mu g$.
↓
retardation.

$\boxed{\mu = \frac{V_0^2}{2L}}$

d) $v^2 = u^2 + 2a\, L/2$.

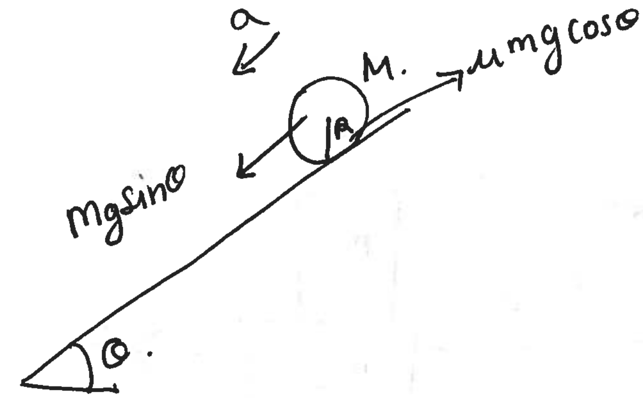
$= V_0/\sqrt{2}$.

e) = Power = force x velociti

$= -\mu mg \times \frac{V_0}{\sqrt{2}}$.

$= -\frac{mV_0^3}{2\sqrt{2}L}$.

4



$$I\alpha = F \cdot R.$$

$$\frac{1}{2}MR^2\alpha = \mu mg\cos\theta\, R \quad -(1)$$

(Since mg sinθ acts at COM it does not appear in torque).

$$\alpha = \frac{a}{R} \quad \text{(No Slipping)}$$

$$\cancel{a \neq g\sin\theta}.$$

$$Ma = Mg\sin\theta - \mu mg\cos\theta.$$

$$Ma = Mg\sin\theta - \frac{1}{2}Ma. \quad \text{(Using (1))}.$$

$$\frac{3}{2}Ma = Mg\sin\theta.$$

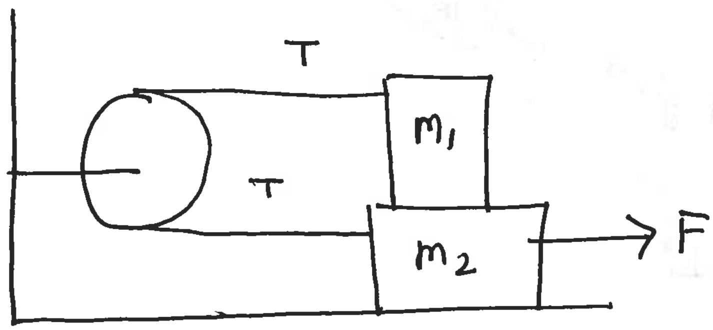
$$a = \frac{2}{3}g\sin\theta. \quad -(2)$$

Plugging it back in (1).

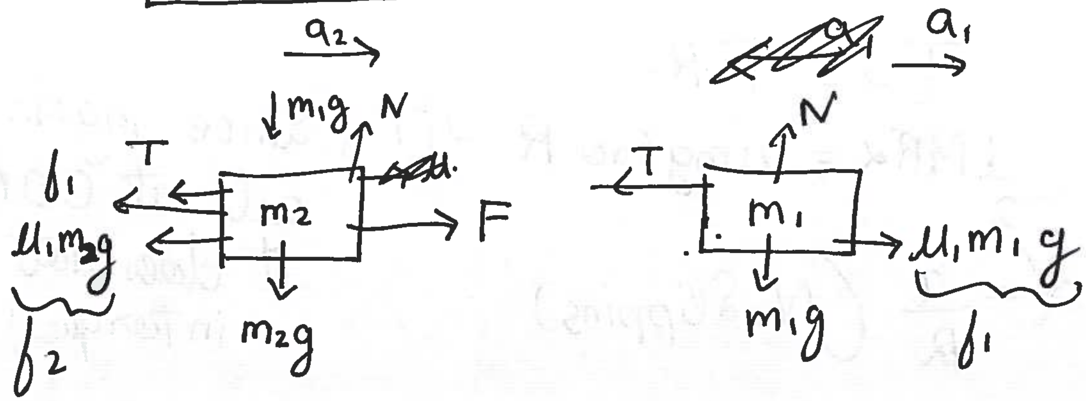
$$\frac{1}{2}MR^2\,\frac{2}{3}\,\frac{g\sin\theta}{R} = \mu mg\cos\theta\, R.$$

$$\Rightarrow \boxed{\frac{\tan\theta}{3} = \mu}$$

6



a)



b) $m_1 a_1 = f_1 - T$

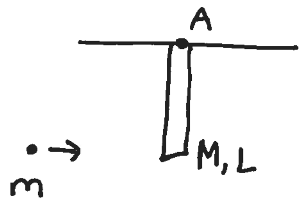
$N_1 - m_1 g = 0.$

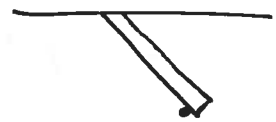
c). $F - T - f_1 - f_2 = m_2 a_2.$

$N_2 - m_1 g - m_2 g = 0.$

d) & e). Using the 4 relations. these can be solved easily.

5





Pivot is point A

$$\boxed{\text{a) } I_{\text{combined}} = \Big(\underbrace{\frac{ML^2}{3}}_{\text{rod about edge}} + \underbrace{mL^2}_{\text{due to ball}} \Big)}$$

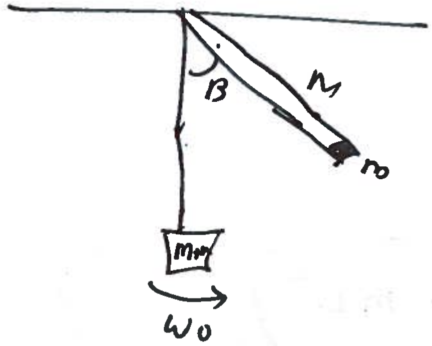
b). $$\left.\begin{array}{l}\text{CM of rod} = L/2. \\ \text{Position of ball} = L\end{array}\right\} \text{ both counting from point A}$$

$$CM_Y = \frac{\sum_i m_i y_i}{\sum m_i} = \underbrace{\frac{mL + ML/2}{M+m}}_{\text{CM position below X.}}$$

CM position from center of rod

$$= \frac{L}{2} - \frac{mL + ML/2}{M+m} = \boxed{\frac{1}{2} mL/(m+M)}$$

c) after the collision both the rod & ball move with angular velocity ω. & rise upto $\angle \beta$.



Initial Energy $= \frac{1}{2} I \omega^2$.

Final Energy = Potential Energy (Height above reference)

$$= \left(m + \frac{M}{2}\right) g L (1 - \cos\beta).$$

$$\frac{1}{2} I \omega^2 = \left(m + \frac{M}{2}\right) g L (1 - \cos\beta).$$

$$\omega = \frac{1}{L}\sqrt{\frac{\left(m+\frac{M}{2}\right) g L (1-\cos\beta)}{\left(m+\frac{M}{3}\right)}}$$

~~$\omega \times R = V \Rightarrow \omega \times L = V = V = \sqrt{\frac{\left(m+\frac{M}{2}\right) g L (1-\cos\beta)}{\left(m+\frac{M}{3}\right)}}$~~.

~~This is the velocity of~~.

This is the $\angle$ular Velocity. after collision.

To find velocity of ball we must conserve Lular Momentum before & After Collision.

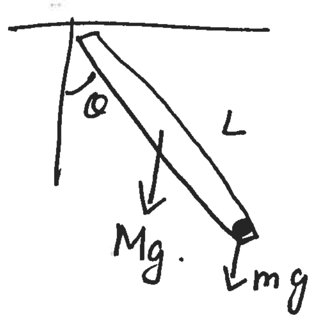
After Collision $= I\omega$.

Before Collision $= mv_0L$. (from $mv \times r$).

$$mv_0L = \frac{\cancel{M}\left(m+\frac{1}{3}M\right)L^2}{L}\sqrt{\left(\frac{m+\frac{M}{2}}{m+\frac{M}{3}}\right)\left(2gL(1-\cos\beta)\right)}$$

$$\Rightarrow \boxed{V_0 = \left(\frac{m+\frac{M}{3}}{m}\right)\sqrt{\left(\frac{m+\frac{M}{2}}{m+\frac{M}{3}}\right)\left(2gL(1-\cos\beta)\right)}}$$

d) Suppose the rod has raised θ.



Torque due to Gravity.

$= mg \times r + Mg \times r'$

$= -\left(mgL\sin\theta + \frac{M}{2}Lg\sin\theta\right)$

for small angle $\sin\theta \simeq \theta$

$$-\left(mgL\theta + \frac{MgL}{2}\theta\right).$$

$$\tau = I\alpha.$$

$$I\alpha = -\left(mgL + \frac{MgL}{2}\right)\theta.$$

$$\ddot{\theta} = -\underbrace{\frac{\left(m+\frac{M}{2}\right)}{\left(m+\frac{M}{3}\right)}\left(\frac{g}{L}\right)}_{k}.$$

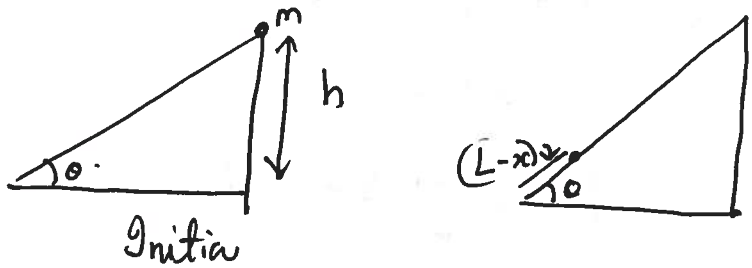
$$\therefore \boxed{f = \frac{1}{2\pi}\sqrt{\frac{m+\frac{M}{2}}{m+\frac{M}{3}}\left(\frac{g}{L}\right)}}$$

7

b)
i) Work done by Spring

$$= -\frac{1}{2}kx^2 \quad [\text{Since it compresses by } x]$$

ii) Work done by Weight.



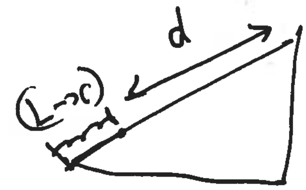
We need to find the vertical distance covered by block.

$$= [h - (L-x)\sin\theta].$$

$$\text{Work} = mg[h - (L-x)\sin\theta].$$

iii) Frictional work → always measured along the ~~Inclide~~ Incline

Total length of Incline = $H/\sin\theta$.

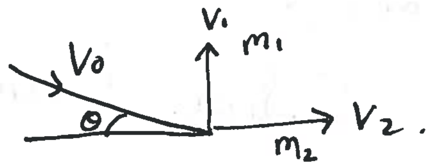


$$W_f = \mu mg\cos\theta \, d.$$

$$= \mu mg\cos\theta\left(\frac{H}{\sin\theta} - (L-x)\right).$$

c). $W_{weight} + W_{spring} + W_{friction} = 0.$

(8) Let Initial velocity be V_0 Inclined at at $\angle\theta$ to x-axis



Momentum Conservation

along $x \Rightarrow$ $\underbrace{(M_1+M_2)}_{\text{Initial mass}} V_0 \cos\theta = M_2 V_2$ — (1)

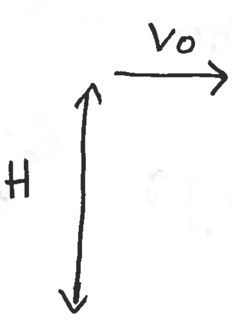
along $y \Rightarrow$ $(M_1+M_2) V_0 \sin\theta = M_1 V_1$ — (2)

$(1)^2 + (2)^2 \Rightarrow (M_1+M_2)^2 V_0^2 = (m_1 V_1)^2 + (m_2 V_2)^2$.

$$\Rightarrow \quad V_0 = \sqrt{\frac{(M_1 V_1)^2 + (m_2 V_2)^2}{(M_1+M_2)}}$$

$(2)/(1)$ $\cancel{(1)/(2)}$ will give the angle

(9)



along x = Const velocity V_0.

along y = Initial velocity = 0

Acceleration = g.

a) $x(t) = V_0 t$

$\cancel{b)}$

$y(t) = H - \frac{1}{2} g t^2$

↳ Initial Height.

b). Angular Momentum wrt origin.

$= m V_0 \times r$

$\cancel{V_x \neq V_y}$

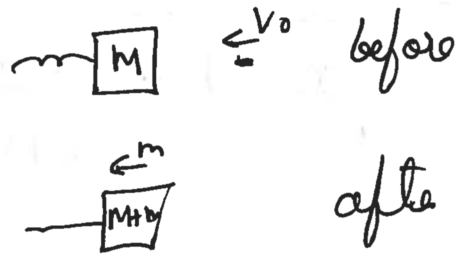
$= \cancel{mV_0 \times} = m V_0 \hat{i} \times (x(t)\hat{i} + y(t)\hat{j})$.

$= -(m V_0 H + \frac{1}{2} m g V_0 t^2)\hat{k}$

c) & d).

$$\boxed{\tau = \frac{dL}{dt}}$$

10 a) Velocity of Block + Bullet after collision.



$$\Rightarrow \quad mV_0 = (M+m)V$$

$$V = \frac{mV_0}{M+m}$$ from Momentum Conservation.

Now we apply Energy conservation on Spring.

$$\frac{1}{2}kx^2 = \underbrace{\frac{1}{2}(M+m)V^2}_{\text{Block + Bullet System}}.$$

$$= \boxed{x = \frac{mV_0}{\sqrt{(M+m)k}}}$$

c) Recall frequency for a spring block system is.

$$\cancel{f = 2\pi\sqrt{\ }} \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

here mass = $M+m$.

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{M+m}}.$$

b). Position as a function of time.

$$x(t) = -x\,\sin(\omega t).$$

Equation for SHM

$$f = 2\pi\omega$$

$$\therefore \boxed{x(t) = \frac{-mv_0}{\sqrt{(M+m)k}} \sin\left(\sqrt{\frac{k}{M+m}}\,t\right).}$$

d). When would the block return to equilibrium.

when $x(t) = 0$.

ie when $\sin\left(\sqrt{\frac{k}{M+m}}\,t\right) = 0$.

$\sin$ is 0 when $\sin(\pi)$.

$$\therefore \boxed{t = \frac{\pi}{\sqrt{\frac{k}{M+m}}}}$$

e). Maximum speed is always at equilibrium

ie $x = 0$.

can also be checked by taking $\frac{dx(t)}{dt}$

11 Initial $I_i = \underset{\text{rod}}{I} + M_1 A^2 + m_2 B^2$

Final $I_f = I + M_1 A^2 + (M_2 + m) B^2$.

Initial Angular momentum $= I_i \omega_0$

Final Angular Momentum =

$$I_f \omega_f + \underbrace{\text{momentum Imparted by bullet}}_{m v_0 B \cos\theta}.$$

from $m v \times r$.

So from Momentum Conservation.

$$I_i \omega_0 = I_f \omega_f + m v_0 B \cos\theta.$$

$$\omega = \left| \frac{m v_0 B \cos\theta - (I + M_1 A^2 + M_2 B^2)}{I + M_1 A^2 + (M_2 + m) B^2} \right|.$$

direction of rotation depends on

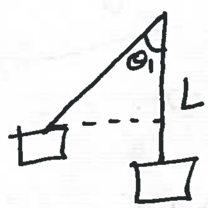
$$m v_0 B \cos\theta \gtrless I + M_1 A^2 + M_2 B^2$$

(12) Since there was explosion ~~we~~ & the boxes fly off in opp. direction we have conservation of momentum.

$$m_1 v_1 = m_2 v_2.$$

$$v_2 = \frac{m_1 v_1}{m_2} \quad \text{—(1)}$$

for m_1 it goes up till θ_1



Its height above reference $= L(1-\cos\theta)$.

from Conservation of Energy.

$$\frac{1}{2} m_1 v_1^2 = mg L(1-\cos\theta_1)$$

$$v_1 = \sqrt{2gL(1-\cos\theta_1)} \quad \text{—(2)}$$

Now we apply conservation to m_2.

$$\frac{1}{2} m_2 v_2^2 = mg L(1-\cos\theta_2).$$

using (1) & (2) we get.

$$\boxed{\theta_2 = \cos^{-1}\left[1 - \frac{m_1^2}{m_2^2}(1-\cos\theta_1)\right].}$$

(13) $\omega = 2\pi f$.

a) $T = \frac{2\pi}{\omega}$ $\quad$ $\underbrace{T = 2\pi\sqrt{\frac{R^3}{GM}}}$.

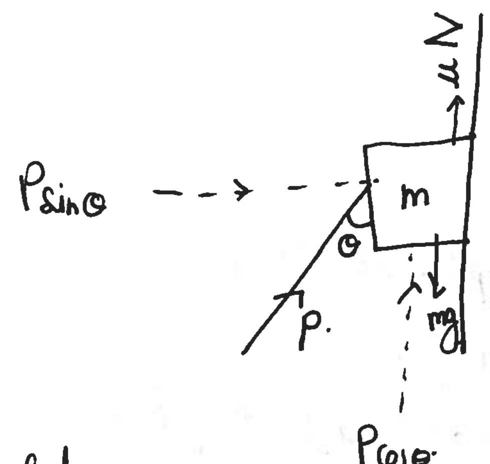
Using this and, you can calculate altitude (ie R).

b) Orbital Speed.

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

plug in the R calculated from a). & find v.

(14)



Normal force $= P\sin\theta$ $\quad$ Friction $= \mu N = \mu P\sin\theta$

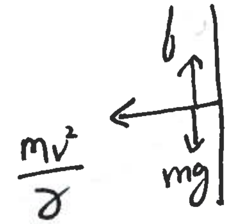
Balancing forces on a vertical direction

$$mg = P\cos\theta + \mu P\sin\theta.$$

$$P = \frac{mg}{\mu_s \sin\theta + \cos\theta}.$$

(15) Centripetal force $= \frac{mv^2}{r}$

friction $= \mu N = \frac{\mu m v^2}{r}$.



Balancing Vertical forces.

$$\frac{\mu m v^2}{R} = mg.$$

$$= \frac{v^2}{R^2} = \frac{g}{\mu R}.$$

$$= \boxed{\omega^2 = \frac{g}{\mu R}}$$

(16) Let Spring be compressed by x.

Initial Energy $= mgh$

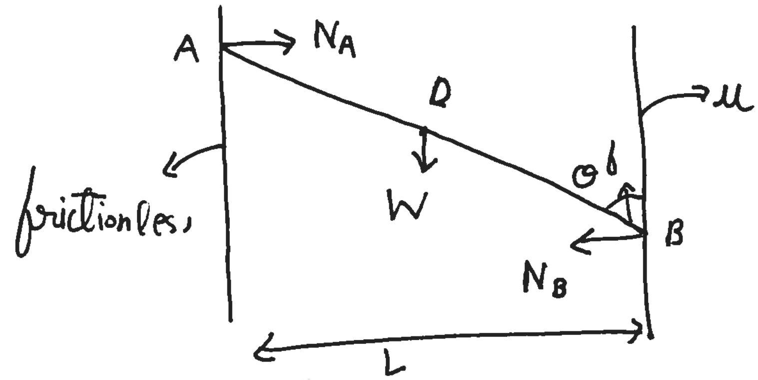
Final Energy $= mg(L-x) + \frac{1}{2}kx^2$.

$$mgh = mg(L-x) + \frac{1}{2}kx^2.$$

Solve the Quadratic wrt x.

$$x = \frac{mg + \sqrt{m^2g^2 + 2kmg(h-L)}}{k}.$$

(19)



$$\sum \tau = 0 = \frac{mg}{2} D\sin\theta - \cancel{N} N_A D\cos\theta = 0.$$

@ B

$$= N_A = \frac{mg}{2}\tan\theta. \quad (1)$$

$N_A = N_B$ from equating forces in x-direction

$$F_f = W \Rightarrow \mu N_B = mg.$$

$$N_B = \frac{mg}{\mu}. \quad (2)$$

$$\frac{mg}{\mu} = \frac{mg}{2}\tan\theta. \Rightarrow \theta = \tan^{-1}\left(\frac{2}{\mu}\right)$$

Length of board

$$D\sin\theta = L.$$

$$\boxed{D = L\sqrt{1+\frac{\mu^2}{4}}.}$$